The world of maths

Progression of teaching NCETM spines in Year 1	
Year 1 Number, addition, and subtraction	
1.1 Comparison of quantities and measures	**Teaching Point 1:** Items can be compared according to attributes such as length (or height or breadth), area, volume/capacity or weight/mass. **Teaching Point 2:** When comparing two sets of objects, one set can contain more objects than the other and one set can contain fewer objects than the other, or both sets can contain the same number of objects. **Teaching Point 3:** The symbols <, > and = can be used to express the relative number of objects in two sets, or the relative size of two numbers.
1.2 Introducing 'whole' and 'parts': part-part-whole	**Teaching point 1:** A 'whole' can be represented by one object; if some of the whole object is missing, it is not the 'whole'. **Teaching point 2:** A whole object can be split into two or more parts in many different ways. The parts might look different; each part will be smaller than the whole, and the parts can be combined to make the whole. **Teaching point 3:** A 'whole' can be represented by a group of discrete objects. If some of the objects in the group are missing, it is not the whole group – it is part of the whole group. **Teaching point 4:** A whole group of objects can be composed of two or more parts, and this can be represented using a part–part–whole 'cherry' diagram. The group can be split in many different ways. The parts might look different; each part will be smaller than the whole group and the parts can be combined to make the whole group.
1.3 Composition of numbers 0-5	**Teaching point 1:** Numbers can represent how many objects there are in a set; for small sets we can recognise the number of objects (subitise) instead of counting them. **Teaching point 2:** Ordinal numbers indicate a single item or event, rather than a quantity. **Teaching point 3:** Each of the numbers one to five can be partitioned in different ways. **Teaching point 4:** Each of the numbers one to five can be partitioned in a systematic way. **Teaching point 5:** Each of the numbers one to five can be partitioned into two parts; if we know one part, we can find the other part.

	Teaching point 6: The number before a given number is one less; the number after a given number is one more. **Teaching point 7:** Partitioning can be represented using the bar model.
1.4 Composition of numbers 6-10	**Teaching point 1:** The numbers six to nine are composed of 'five and a bit'. Ten is composed of five and five. **Teaching point 2:** Six, seven, eight and nine lie between five and ten on a number line. **Teaching point 3:** Numbers that can be made out of groups of two are even numbers; numbers that can't be made out of groups of two are odd numbers. Even numbers can be partitioned into two odd parts or two even parts; odd numbers can be partitioned into one odd part and one even part. **Teaching point 4:** Each of the numbers six to ten can be partitioned in different ways. The numbers six to ten can be partitioned in a systematic way. **Teaching point 5:** Each of the numbers six to ten can be partitioned into two parts; if we know one part, we can find the other part.
1.5 Additive structures; introduction to aggregation and partitioning	**Teaching point 1:** combining two or more parts to make a whole is called aggregation; the addition symbol, +, can be used to represent aggregation. **Teaching point 2:** The equals symbol, =, can be used to show equivalence between the whole and the sum of the parts. **Teaching point 3:** Each addend represents a part, and these are combined to form the whole/sum; we can find the value of the whole by adding the parts. We can represent problems with missing parts using an addition equation with a missing addend. **Teaching point 4:** Breaking a whole down into two or more parts is called partitioning; the subtraction symbol, −, can be used to represent partitioning.
1.6 Additive structures; introduction to augmentation and reduction	**Teaching point 1:** An addition context described by a *'first…, then…, now…'* story is an example of augmentation. We can link the story to a numerical representation – each number represents something in the story. **Teaching point 2:** A subtraction context described by a *'first…, then…, now…'* story is an example of reduction. We can link the story to a numerical representation – each number represents something in the story. **Teaching point 3:** Given any two parts of the story we can work out the third part; given any two numbers in the equation we can find the third one. **Teaching point 4:** Addition and subtraction are inverse operations.
1.7 Addition and subtraction:	**Teaching point 1**: Addition is commutative: when the order of the addends is changed, the sum remains the same. **Teaching point 2**: Ten can be partitioned into pairs of numbers that sum to ten. Recall of these pairs of numbers supports calculation.

strategies within 10	**Teaching point 3**: Adding one gives one more; subtracting one gives one less. **Teaching point 4**: Consecutive numbers have a difference of one; we can use this to solve subtraction equations where the subtrahend is one less than the minuend. **Teaching point 5**: Adding two to an odd number gives the next odd number; adding two to an even number gives the next even number. Subtracting two from an odd number gives the previous odd number; subtracting two from an even number gives the previous even number. **Teaching point 6**: Consecutive odd / consecutive even numbers have a difference of two; we can use this to solve subtraction equations where the subtrahend is two less than the minuend. **Teaching point 7**: When zero is added to a number, the number remains unchanged; when zero is subtracted from a number, the number remains unchanged. **Teaching point 8**: Subtracting a number from itself gives a difference of zero. **Teaching point 9**: Doubling a whole number always gives an even number and can be used to add two equal addends; halving is the inverse of doubling and can be used to subtract a number from its double. Memorised doubles/halves can be used to calculate near-doubles/halves. **Teaching point 10**: Addition and subtraction facts for the pairs five and three, and six and three, can be related to known facts and strategies.
1.8 Composition of numbers: 20-100	**Teaching point 1**: There is a set counting sequence for counting to 100 and beyond. **Teaching point 2**: Objects can be counted efficiently by making groups of ten. The digits in the numbers 20–99 tell us about their value. **Teaching point 3**: Each number on the 0–100 number line has a unique position. **Teaching point 4**: The relative size of two two-digit numbers can be determined by first examining the tens digits and then, if necessary, examining the ones digits, with reference to the cardinal or ordinal value of the numbers. **Teaching point 5**: Each two-digit number can be partitioned into a tens part and a ones part. **Teaching point 6**: The tens and ones structure of two-digit numbers can be used to support additive calculation.
1.9 Composition of numbers: 20-100	**Teaching point 1**: There is a set counting sequence for counting to 100 and beyond. **Teaching point 2**: Objects can be counted efficiently by making groups of ten. The digits in the numbers 20–99 tell us about their value. **Teaching point 3**: Each number on the 0–100 number line has a unique position. **Teaching point 4**: The relative size of two two-digit numbers can be determined by first examining the tens digits and

	then, if necessary, examining the ones digits, with reference to the cardinal or ordinal value of the numbers. **Teaching point 5**: Each two-digit number can be partitioned into a tens part and a ones part. **Teaching point 6**: The tens and ones structure of two-digit numbers can be used to support additive calculation.
1.10 Composition of numbers:11-19	**Teaching point 1**: The digits in the numbers 11–19 tell us about their value. **Teaching point 2**: The numbers 11–19 can be formed by combining a ten and ones and can be partitioned into a ten and ones. **Teaching point 3**: A number is even if the ones digit is even; it *can* be made from groups of two. A number is odd if the ones digit is odd; it *can't* be made from groups of two. **Teaching point 4**: Doubling the numbers 6–9 (inclusive) gives an even teen number; halving an even teen number gives a number from six to nine (inclusive). **Teaching point 5**: Addition and subtraction facts within 10 can be applied to addition and subtraction within 20.
Multiplication and division	
2.1 Counting, unitising, and coins	**Teaching point 1**: We can count efficiently by counting in groups of two. **Teaching point 2**: We can count efficiently by counting in groups of ten. **Teaching point 3**: We can count efficiently by counting in groups of five. **Teaching point 4**: A coin has a value which is independent of its size, shape, colour, or mass. **Teaching point 5**: The *number* of coins in a set is different from the *value* of the coins in a set; knowledge of counting in groups of two, five or ten can be used to work out the value of a set of identical low-denomination coins. **Teaching point 6**: Knowledge of counting in groups of two, five or ten can be used to work out how many identical low-denomination coins are needed to make a given value.
Geometry	Properties of shape Pupils will be taught to: • recognise and name common 2-D and 3-D shapes, including: • 2-D shapes [for example, rectangles (including squares), circles and triangles] • 3-D shapes [for example, cuboids (including cubes), pyramids and spheres]. Position and direction Pupils should be taught to: describe position, direction and movement, including whole, half, quarter and three-quarter turns.
Measurement	Pupils will be taught to:

	• compare, describe and solve practical problems for: • lengths and heights [for example, long/short, longer/shorter, tall/short, double/half] • mass/weight [for example, heavy/light, heavier than, lighter than] • capacity and volume [for example, full/empty, more than, less than, half, half full, quarter] • time [for example, quicker, slower, earlier, later] • measure and begin to record the following: • lengths and heights • mass/weight • capacity and volume • time (hours, minutes, seconds) • recognise and know the value of different denominations of coins and notes • sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening]

Progression of teaching NCETM spines in Year 2	
Year 2 Number, addition, and subtraction	
1.11 Addition and subtraction: bridging 10	**Teaching point 1**: Addition of three addends can be described by an aggregation story with three parts. **Teaching point 2**: Addition of three addends can be described by an augmentation story with a ***'first…, then…, then…, now…'*** structure. **Teaching point 3**: The order in which addends (parts) are added or grouped does not change the sum (associative and commutative laws). **Teaching point 4**: When we are adding three numbers, we choose the most efficient order in which to add them, including identifying two addends that make ten (combining). **Teaching point 5**: We can add two numbers which bridge the tens boundary by using a 'make ten' strategy. **Teaching point 6**: We can subtract across the tens boundary by subtracting *through* ten or subtracting *from* ten.
1.12 Subtraction as difference	**Teaching point 1**: Difference compares the number of objects in one set with the number of objects in another set; or the difference between two measures. **Teaching point 2**: Difference is one of the structures of subtraction. **Teaching point 3**: Consecutive whole numbers have a difference of one; consecutive odd/even numbers have a difference of two. **Teaching point 4**: We can apply the structure of difference to compare data.
1.13 Addition and Subtraction: Two digit and single digit numbers	**Teaching point 1**: Knowledge of the number line, and quantity values of numbers, can be applied to add/subtract one to/from a given two-digit number. **Teaching point 2**: Known facts for the numbers *within* ten can be applied to addition/subtraction of a single-digit number to/from a two-digit number. **Teaching point 3**: Knowledge of numbers which sum to ten can be applied to the addition of a single-digit number and two-digit number that sum to a multiple of ten, or subtraction of a single-digit number from a multiple of ten. **Teaching point 4**: Known strategies for addition or subtraction bridging ten can be applied to addition or subtraction bridging a multiple of ten.
1.14 Addition and subtraction: Two -digit numbers	**Teaching point 1**: When finding ten more or ten less than any two-digit number, the ones digit does not change. **Teaching point 2**: When ten is added or subtracted to/from a two-digit number, the tens digit changes and the ones digit stays the same. **Teaching point 3**: Knowledge of number facts within ten can be applied to adding or subtracting multiples of ten

and multiples of ten	to/from a two-digit number. **Teaching point 4**: Two-digit numbers can be partitioned in different ways.
1.15 Addition: Two-digit and two-digit numbers	**Teaching point 1**: Known strategies can be combined to add two multiples of ten to two single-digit numbers. **Teaching point 2**: Two two-digit numbers can be added by partitioning one or both of them into tens and ones. *(This can be done after multiplication and division spines)*
Multiplication and division	
2.2 Structures: Multiplication as equal groups	**Teaching point 1**: Objects can be grouped into equal or unequal groups. **Teaching point 2**: When describing equally grouped objects, the number of groups and the size of the groups must both be defined. **Teaching point 3**: Equal groups can be represented with a repeated addition expression. **Teaching point 4**: Equal groups can be represented with a multiplication expression. **Teaching point 5**: Multiplication expressions can be written for cases where the groups each contain zero items, and for cases where the groups each contain one item.
2.3 Times tables: groups of 2 and commutativity (part 1)	**Teaching point 1**: For equally grouped objects, the number of groups is a factor, the group size is a factor, and the overall number of objects is the product; this can be represented with a multiplication equation. Counting in multiples of two can be used to find the product when the group size is two. **Teaching point 2**: Counting in multiples of two can be represented by the two times table. Adjacent multiples of two have a difference of two. Facts from the two times table can be used to solve problems about groups of two. **Teaching point 3**: Factor pairs can be written in either order, with the product remaining the same (commutativity).
2.4 Times tables: groups of 10, and 5 and factors 0 and 1	**Teaching point 1**: Counting in multiples of ten can be represented by the ten times table. Adjacent multiples of ten have a difference of ten. Facts from the ten times table can be used to solve problems about groups of ten. **Teaching point 2**: Counting in multiples of five can be represented by the five times table. Adjacent multiples of five have a difference of five. Facts from the five times table can be used to solve problems about groups of five. **Teaching point 3**: Skip counting, and grouping can be used to explore the relationship between the five times table and the ten times table.

	Teaching point 4: When zero is a factor, the product is zero. When one is a factor, the product is equal to the other factor (if there are only two factors).
2.5 Commutativity (part 2), doubling and halving	**Teaching point 1**: The same multiplication equation can have two different grouping interpretations. Problems about two/five/ten equal groups can be solved using facts from the two/five/ten times table. (commutativity) **Teaching point 2**: If two is a factor, knowledge of doubling facts can be used to find the product; problems about doubling can be solved using facts from the two times table. **Teaching point 3**: Halving is the inverse of doubling; problems about halving can be solved using facts from the two times table and known doubling facts. **Teaching point 4**: Products in the ten times table are double the products in the five times table; products in the five times table are half of the products in the ten times table.
2.6 Structures: quotative and partitive division	**Teaching point 1**: Objects can be grouped equally, sometimes with a remainder. **Teaching point 2**: Division equations can be used to represent 'grouping' problems, where the total quantity (dividend) and the group size (divisor) are known; the number of groups (quotient) can be calculated by skip counting in the divisor. (quotitive division) **Teaching point 3**: Division equations can be used to represent 'sharing' problems, where the total quantity (dividend) and the number we are sharing between (divisor) are known; the size of the shares (quotient) can be calculated by skip counting in the divisor. (partitive division) **Teaching point 4**: Strategies for finding the quotient, that are more efficient than skip counting, include using known multiplication facts and, when the divisor is two, using known halving facts. **Teaching point 5**: When the dividend is zero, the quotient is zero; when the dividend is equal to the divisor, the quotient is one; when the divisor is equal to one, the quotient is equal to the dividend.
Addition and subtraction	
1.15 Addition: Two-digit and two-digit numbers	**Teaching point 1**: Known strategies can be combined to add two multiples of ten to two single-digit numbers. **Teaching point 2**: Two two-digit numbers can be added by partitioning one or both of them into tens and ones. *(This can be taught after multiplication/division spines or recapped if taught before to check understanding.)*

1.16 Subtraction: two-digit, two-digit	**Teaching point 1**: Known strategies can be used to subtract a multiple of ten and a single-digit number from a two-digit number. **Teaching point 2**: A two-digit number can be subtracted from a two-digit number by partitioning the subtrahend into tens and ones.
Fractions	• 1. Name the fractions 'one-half', 'one-quarter' and 'one-third' in relation to a fraction of a length, shape or set of objects. • **2**: Read and write the fraction notation 12, 13 and 14, and relate this to a fraction of a length, shape or set of objects. • **3**: Find half of numbers. • **4**: Find 13 or 14 of a number. • **5**: Find 24 and 34 of an objects, shape, set of objects, length or quantity; recognise the equivalence of 24 and 12.
Geometry	Properties of shape Pupils will be taught to: • identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line • identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces • identify 2-D shapes on the surface of 3-D shapes, [for example, a circle on a cylinder and a triangle on a pyramid] • compare and sort common 2-D and 3-D shapes and everyday objects. Position and direction Pupils will be taught to: • order and arrange combinations of mathematical objects in patterns and sequences use mathematical vocabulary to describe position, direction, and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anticlockwise).

Measurement	• choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels • compare and order lengths, mass, volume/capacity and record the results using >, < and = • recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value • find different combinations of coins that equal the same amounts of money • solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change • compare and sequence intervals of time • tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times • know the number of minutes in an hour and the number of hours in a day.